Automn School

Macroscopic traffic flow models on networks

PAOLA GOATIN
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The lecture will address the following topics:

- Macroscopic traffic flow models (first and second order)
- Basics on conservation laws: characteristics method, weak solutions, entropy condition, Riemann problem
- Conservation laws on networks: the Riemann problem at junctions
- Finite volume schemes (Godunov)

A short introduction to periodic homogenization

DANIEL ONOFREI
Departement of Mathematics – University of Houston

Almost all the materials available in nature have some degree of heterogeneity, that is, they contain an underlying microstructure. So, determining and understanding the inner heterogenous structure in a given material is important for the determination of its properties and it subsequent use.

A different but equally important problem is, given a prescribed microstructure, to determine its various properties (electric or heat conductivity, resistance, permittivity, permeability, etc.) at the macro scale. A good understanding of this question leads to the knowledge off how small changes at the microscale (scale of the underlying microstructure) affect the macro scale behavior and this in turn leads to smart ways to optimize desired effects.

In this 7 lectures we will study this later question and focus on the concept of periodic microstructures (i.e. materials which are constructed through a periodic repetition of a given small scale mixture, e.g. brick wall or a laminate wood). We will start with a brief introduction of the concept and then study the main question above in the context of the toy problem of understanding the macro behavior of a one dimensional periodic microstructure. This problem can be seen as a model for the steady state heat flow through a long microstructured bar. Then we will briefly extend the limit analysis to the problem of layered media.

Then, for more general periodic microstructures we will introduce first the method of asymptotic expansion for an heuristic derivation of the limit behavior and then present the method of periodic unfolding for a rigorous analysis.

The last lecture will be dedicated to presenting several interesting open questions.

Numerical optimization of partial differential equations

BARTEK PROTAS
Department of Mathematics Statistics – McMaster University

The objective of these lectures is to introduce students to the main concepts and practice of optimization applied to systems described by partial differential equations (PDEs). Such problems arise in diverse areas of science and engineering, including fluid and solid mechanics, thermodynamics, heat transfer and electromagnetism. Since problems described by PDEs are typically defined in infinite-dimensional spaces, they are often not amenable to standard optimization techniques designed for finite-dimensional problems. This, in particular, concerns the computation of gradients (sensitivities) of objective functions accounting for the PDE constraints, which is typically done using the adjoint calculus. In the lectures, we will

1. introduce basic concepts of nonlinear optimization, such as gradient descent, line minimization, the method of conjugate gradients, etc.,

2. show now gradient information can be conveniently obtained for PDE problems using the adjoint calculus; we will also discuss how regularity of solutions can be enforced by using Sobolev gradients,

3. demonstrate how the tools from parts (i) and (ii) above can be combined to design state-of-the-art algorithms for the solution of PDE optimization problems.

Time permitting, we may also discuss additional topics such as regularization and shape optimization. The lectures will address both theoretical and algorithmic aspects, with a number of computational examples illustrating the implementation of different algorithms.

Conference

A New Perilous Playground for Viscosity Solutions: Problems with Discontinuities

GUY BARLES
Institut Denis Poisson – Université de Tours

Recently a lot of works have been devoted to the study of Hamilton-Jacobi Equations either in the case when they involve discontinuities or when they are set on networks. Surprisingly, these two types of problems share common features. The aim of this talk is to try to describe the kind of results which can be obtained and the the main ideas which are used in the simplest possible framework, namely the Hamilton-Jacobi analogue of scalar conservation laws with discontinuous flux.

Parametric shape optimization using the support function

BENIAMIN BOGOSEL
CMAP – École Polytechnique

The numerical study of shape optimization problems under convexity, diameter or constant width constraints is difficult. The source of the difficulties is the non-local character of such constraints. The class of admissible perturbations is restricted in regions where the constraint is saturated. In this presentation I will present a method, based on the support function associated to a convex set, which allows to overcome such difficulties. The constraints are transformed into algebraic inequalities on coefficients of a certain spectral decomposition of the support function. This allows the use of standard optimization algorithms in order to approximate solutions of a large family of shape optimization problems.

The problems considered deal with functionals involving the volume, the perimeter and the eigenvalues of the Dirichlet-Laplace operator under the various constraints recalled above. In particular, the Meissner conjecture regarding the bodies of constant width of minimal volume in dimension three is confirmed numerically by solving a discrete optimization problem. This work is in collaboration with Pedro Antunes.

Minimal k-partition for the p-norm of the eigenvalues

VIRGINIE BONAILLIE-NOËL
CNRS

In this talk,we analyze the connections between the nodal domains of the eigenfunctions of the Dirichlet-Laplacian and the partitions of the domain by k open sets D_i which are minimal in the sense that the maximum over the D_i's of the groundstate energy of the Dirichlet realization of the Laplacian is minimal. Instead of considering the maximum among the first eigenvalues, we can also consider the p-norm of the vector composed by the first eigenvalues of each subdomain.

Partially congested propagation fronts in a 1d compressible Navier-Stokes model

ANNE-LAURE DALIBARD
LJLL, UPMC

The purpose of this talk is to present some results on the asymptotic behavior of a 1d Navier-Stokes system with a singular pressure accounting for soft congestion effects. We prove the existence of propagation fronts for this model, with congested and non-congested zones. We also investigate the properties of these partially congested propagation fronts: existence and uniqueness of strong solutions in the vicinity of such profiles, asymptotic stability. This is a joint work with Charlotte Perrin.

Hamilton-Jacobi equations for optimal control on network with entry costs

KHANG DAO
KTH, Stockholm

We consider an optimal control problem on a network in which there are entry (or exit) costs at each edge. The effect of the switching costs is to make the value function discontinuous at the vertex. Therefore, the first problem is to determine the appropriate Hamilton-Jacobi equations (especially at the vertex) and prove that our value function is a viscosity solution of these equations. Secondly, we propose two proofs of a comparison principle based on arguments from the theory of optimal control which follows the ideas introduced by Achdou, Oudet and Tchou (2015) and PDEs techniques inspired by Lions and Souganidis (2016). Finally, from comparison principle, we obtain that our value function is a unique solution of Hamilton-Jacobi equations.

Shape optimization of a layer by layer mechanical constraint for additive manufacturing

CHARLES DAPOGNY
Laboratoire Jean Kuntzmann – Université Grenoble Alpes

In this presentation, we introduce a new constraint functional for shape optimization problems, which enforces the constructibility by means of additive manufacturing processes, and helps in preventing the appearance of overhangs – large regions hanging over void which are notoriously difficult to assemble using such technologies.

The proposed constraint relies on a simplified model for the construction process: it involves a continuum of shapes, namely the intermediate shapes corresponding to the stages of the construction process where the total, final shape is erected only up to a certain level.

The shape differentiability of this constraint functional is analyzed - which is not a standard issue because of its peculiar structure. Several numerical strategies and examples are then presented.

Enriched homogenization in presence of boundaries or interfaces

SONIA FLISS
UMA – ENSTA

This work is motivated by the fact that classical homogenization theory poorly takes into account interfaces or boundaries. It is particularly unfortunate when one is interested in phenomena arising at the interfaces or the boundaries of the periodic media (the propagation of plasmonic waves at the surface of metamaterials for instance). To overcome this limitation, we have constructed an effective model which is enriched near the interfaces and the boundaries. For now, we have treated and analysed the case of simple geometries : for instance a plane interface between a homogeneous and a periodic half spaces. We have derived a high order approximate model which consists in replacing the periodic media by an effective one but the transmission conditions are not classical. The obtained conditions involve Laplace- Beltrami operators at the interface and requires to solve cell problems in periodicity cell (as in classical homogenization) and in infinite strips (to take into account the phenomena near the interface). We establish well posedness for the approximate model and error estimates which justify that this new model is more accurate near the interface and in the bulk. From a numerical point of view, the only difficulty comes from the problems set in infinite strips (one half is homogeneous and the other is periodic). This is overcome using Dirichlet-to-Neumann operators corresponding to the homogeneous and the periodic media.The numerical results confirm the theoretical ones.

Conservation laws with local constraints arising in traffic modeling

PAOLA GOATIN
INRIA Sophia Antipolis-Méditerranée

We present a well-posedness result for conservation laws with fixed local constraints on the flux, which are used to model the presence of toll gates or construction sites hindering traffic flow. We then extend the results to moving bottlenecks. More precisely, we consider the Cauchy problem for a strongly coupled PDE-ODE system modeling the influence of a slow vehicle on the surrounding road traffic.

The model consists of a conservation law describing the main traffic evolution and an ODE accounting for the trajectory of the slower vehicle that depends on the downstream traffic density. The moving constraint is operated by an inequality on the flux, which accounts for the bottleneck created on the road by the presence of the slower vehicle.

We present the proof of existence of weak entropy solutions obtained via the wave-front tracking method, and a finite volume scheme able to capture exactly the non-classical discontinuities that may arise at the constraint position.

Quantitative parabolic regularity à la De Giorgi

JESSICA GUERAND
DMA – ENS Paris

De Giorgi method is a way to prove Hölder regularity of solutions of parabolic equations. While in the elliptic case the proof is completely quantitative, in the parabolic case it seems to remain a non-quantitative step: the intermediate value lemma. The purpose of this talk is to present a quantitative version of this step after introducing how it is useful to get Hölder regularity.

Asymptotic analysis of a multiscale parabolic problem with a rough fast oscillating interface

DANIEL ONOFREI
Departement of Mathematics – University of Houston

In this talk we will discuss the well posedness and homogenization for a multiscale parabolic problem in a cylinder Q subset of the whole space.

A rapidly oscillating non-smooth interface inside Q separates the cylinder in two heterogeneous connected components. The interface has a periodic microstructure, and it is situated in a small neighborhood of a hyperplane which separates the two components of Q. The problem models a time-dependent heat transfer in two heterogeneous conducting materials with an imperfect contact between them. At the interface, we assume that the flux is continuous and that the jump of the solution is proportional to the flux. On the exterior boundary, homogeneous Dirichlet boundary conditions are prescribed.

We will show well posedness and discuss the limit analysis through the homogenized problem and relevant corrector results.

A weighted anisotropic Sobolev type inequality and some consequences

GIOVANNI PISANTE
Dipartimento di matematica e fisica – Università della Campania Luigi Vanvitelli

Aim of the talk is to present the anisotropic version of a Sobolev type inequality that involves as weight in the integral norms appropriate powers of the distance function to the boundary. The inequality is sufficiently general to be applied to show the validity of Hardy-Sobolev, Hardy-Morrey and Hardy-John-Niremberg type inequality in the Finsler setting.

Extrem vortex states and the hydrodynamic blow-up problem

BARTEK PROTAS
Department of Mathematics Statistics – McMaster University

In the presentation we will discuss our research program concerning the study of extreme vortex events in viscous incompressible flows. These vortex states arise as the flows saturating certain fundamental mathematical estimates, such as the bounds on the maximum enstrophy growth in 3D (Lu & Doering, 2008). They are therefore intimately related to the question of singularity formation in the 3D Navier-Stokes system, known as the hydrodynamic blow-up problem. We demonstrate how new insights concerning such questions can be obtained by formulating them as variational PDE optimization problems which can be solved computationally using suitable discrete gradient flows. In offering a systematic approach to finding flow solutions which may saturate known estimates, the proposed paradigm provides a bridge between mathematical analysis and scientific computation. In particular, it allows one to determine whether or not certain mathematical estimates are "sharp", in the sense that they can be realized by actual vector fields, or if these estimates may still be improved.

In the presentation we will review a number of results concerning 2D and 3D flows characterized by the maximum possible growth of, respectively, palinstrophy and enstrophy. It will be shown that certain types of initial data, such as the Taylor-Green vortex, which have been used in numerous computational studies of the blow-up problem are in fact a particular instance (corresponding to an asymptotic limit) of our family of extreme vortex states. We will present results comparing the growth of relevant quantities in high-resolution direct numerical simulations of the Navier-Stokes system obtained using our extreme vortex states and different initial data employed in earlier studies. Since none of the 3D computations reveals any tendency for the enstrophy to become unbounded in finite time, the main conclusion is that, should finite-time blow-up be indeed possible in the 3D Navier-Stokes system, it is unlikely to arise from initial data maximizing the instantaneous growth of enstrophy.

Based on a joint work with Diego Ayala and Dongfang Yun.

Particle resolved simulation of suspensions with parallel Lattice Boltzmann Methods

ULRICH RÜDE
Department of Computer Science – Universitaet Erlangen-Nuernberg

The Lattice Boltzman Method (LBM) has emerged as an alternative to simulate fluid flow. Algorithmically, the LBM is an explicit time stepping method that exhibits good scalability so that it is especially suitable when fine resolutions and large meshes are required. Here we will report extensions that permit to simulate submersed particles and their interaction with the fluid. Even when the geometric shapes are resolved, it is possible to model suspensions with a large number of particles. Special attention will be given to the validation of the models and the performance analysis of the parallel execution. Based on this, we will discuss the potential and the limitations of particle-resolved models using the LBM.

Homogenization in thin domains with a rough boundary

MANUEL VILLANUEVA-PESQUEIRA
Departamento de Matemática Aplicada – Universidad Pontificia Comillas

We analyze the behavior of solutions of certain elliptic problems with Neumann boundary conditions posed in thin domains which present a highly oscillatory behavior at their boundaries.

We will consider two-dimensional thin domains with non-smooth periodic oscillatory boundaries and, beyond the periodic case, we consider locally periodic oscillatory boundaries and thin domains where the top and the bottom boundary present oscillations with different profile and different order of frequency.

We are particularly interested in understanding how the geometry of the domain affects the behavior of the solutions. Hence, we obtain the homogenized limit problem as the thickness tends to zero and provide some corrector results for different kind of oscillatory behaviors.

Based on joint work with José M. Arrieta

Unstable waves in kinetic traffic models

GIUSEPPE VISCONTI
Institut für Geometrie und Praktische Mathematik – RWTH Aachen University

We will describe the onset of instabilities, such as stop and go waves, from the point of view of kinetic models for traffic flow. Stop and go waves can be viewed as non equilibrium phenomena which arise in the congested phase of traffic. These waves propagate backwards with respect to the microscopic velocities of individual vehicles.

We show that standard kinetic models exhibit instabilities in asymptotic expansions, signaled by the presence of a non positive diffusion coefficient in the Chapman-Enskog expansion around equilibrium states. The problem becomes: are these instabilities really bad news?

Based on joint work with M. Herty, G. Puppo.